A DIVERGENCE MODEL STUDY OF VARIATIONS FOR DISCRETE FUZZY DISTRIBUTIONS

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ABSTRACT

In the field of information measures, the probability distribution divergence models play a crucial role. On the other hand, fuzzy distributions can be used to solve problems where probability distributions fail. Therefore, it is impossible to ignore the magnitude of hazy events in genuine existence scenarios. The concept of weighted measures and fuzzy distributions are both taken into consideration in the current research, which also includes a study of dissimilar variations in divergence models.

Keywords: Distance measure, Fuzzy events, Fuzzy divergence models, Weighted divergence models, Cross entropy.

INTROTUCTION

It has been established that the sense of distance is absolutely necessary for its applications in the fields of science and engineering. The divergence model, often known as the primary functional Kullback and Leibler's [10] distance model in probability spaces, is given by

$$D(P:Q) = \sum_{i=1}^{n} p_i \ln \frac{p_i}{q_i} \tag{1.1}$$

Researchers from time to time formulated abundant divergence models as follows:

Renyi's [17] model:

$$D_\alpha(P:Q) = \frac{1}{\alpha - 1} \ln \sum_{i=1}^{n} p_i^{\alpha} q_i^{1-\alpha}, \ \alpha \neq 1, \alpha > 0 \tag{1.2}$$

Havrada and Charvat's [5] model:

$$D^\alpha(P:Q) = \frac{1}{\alpha - 1} \left[\sum_{i=1}^{n} p_i^{\alpha} q_i^{1-\alpha} - 1 \right], \alpha \neq 1, \alpha > 0 \tag{1.3}$$

Ferreri's [4] model:

$$D_\lambda(P:Q) = \frac{1}{\lambda} \sum_{i=1}^{n} (1 + \lambda p_i) \ln \frac{1 + \lambda p_i}{1 + \lambda q_i}, \lambda > 0 \tag{1.4}$$

Kapur's [8] models:

$$D_1(P:Q)=\sum_{i=1}^{n} p_i \ln\frac{p_i}{q_i}-\frac{1}{a}\sum_{i=1}^{n}(q_i+ap_i)\ln\frac{(q_i+ap_i)}{q_i(1+a)} \quad (1.5)$$

$$D_2(P:Q)=\frac{1}{\alpha-1}\left[\left(\sum_{i=1}^{n} p_i^{\alpha} q_i^{1-\alpha}-1+A\left[(q_i+ap_i)^{\alpha} q_i^{1-\alpha}-(q_i+ap_i)\right]-A(1+a)^{\alpha}+A(1+a)\right)\right] \quad (1.6)$$

$$D^3(P:Q)=\sum_{i=1}^{n} p_i \ln\frac{p_i}{q_i}-\frac{1}{a}\sum_{i=1}^{n}(1+ap_i)\ln\frac{(1+ap_i)}{1+aq_i};\ a\geq -1 \quad (1.7)$$

$$D_4(P:Q)=\frac{1}{\alpha-1}\ln\left(\sum_{i=1}^{n} p_i^{\alpha+\beta-1} q_i^{1-\alpha}\Big/\sum_{i=1}^{n} p_i^{\beta}\right) \quad (1.8)$$

Taneja's [19] model:

$$T(P:Q)=\frac{1}{2}\sum_{i=1}^{n}\left(\frac{p_i+q_i}{2}\right)\log\left(\frac{p_i+q_i}{2\sqrt{p_i q_i}}\right) \quad (1.9)$$

The above model is also known as arithmetic-geometric mean divergence.

Cichocki and Amari's [3] models:

$$D_A(P:Q)=\frac{1}{\alpha(\alpha-1)}\sum_{i=1}^{n}\left(p_i^{\alpha} q_i^{1-\alpha}-\alpha p_i+(\alpha-1)q_i\right),\ \alpha\neq 0,1 \quad (1.10)$$

and

$$D_B(P:Q)=\sum_{i=1}^{n} p_i\left(\frac{p_i^{\beta-1}-q_i^{\beta-1}}{\beta-1}-\frac{p_i^{\beta}-q_i^{\beta}}{\beta}\right),\beta\neq 0,1 \quad (1.11)$$

The models (1.10) and (1.11) are also known as α -divergence and β -divergence respectively. Some current developments regarding the investigations of divergence models in probability spaces have been made by Lacker [12], Ararat, Hamel and Rudloff [1], Watson, Nieto-Barajas and Holmes [20], Pinelis [16], Sankaran, Sunoj and Nair [18] etc.

On the other hand, a commanding instrument for inaccurate and indistinguishable circumstances where authentic examination is either complicated or unworkable is provided by the theory of fuzzy sets. Zadeh [21] developed this theory and Bhandari and Pal [2] twisted the quantitative model for it. With comparative magnitude W_i, Kapur [9] took the expression for weighted divergence model.

Observing the magnitude of events and the weighted conception, many researchers have twisted abundant weighted divergence models. Kapur [9] made meticulous investigations of the divergence measures in probability spaces and developed numerous fuzzy divergence models for Harvada and Charvat [5], Renyi [17], Ferreri [4] etc. Scores of supplementary such models are comprised of Parkash [14], Parkash and Sharma [15], Joshi and Kumar [6, 7], Markechová, Mosapour and Ebrahimzadeh [13], Kobza [11] etc.

2.1 VARIATIONS IN DIVERGENCE MODELS-A STUDY

For this study, we let

$$g_w(\lambda)=\lambda\sum_{i=1}^{n} w_i\left[\mu_A(x_i)\log\mu_A(x_i)+(1-\mu_A(x_i))\log(1-\mu_A(x_i))\right]$$

$$+(1-\lambda)\sum_{i=1}^{n} w_i\left[\mu_B(x_i)\log\mu_B(x_i)+(1-\mu_B(x_i))\log(1-\mu_B(x_i))\right]$$

$$-\sum_{i=1}^{n} w_i\left[\{\lambda\mu_A(x_i)+(1-\lambda)\mu_B(x_i)\}\log\{\lambda\mu_A(x_i)+(1-\lambda)\mu_B(x_i)\}\right] \tag{2.1}$$

$$+\sum_{i=1}^{n} w_i\left[\{\lambda(1-\mu_A(x_i))+(1-\lambda)(1-\mu_B(x_i))\}\log\{\lambda(1-\mu_A(x_i))+(1-\lambda)(1-\mu_B(x_i))\}\right]$$

So that $g_w(\lambda)$ is a concave function of λ.

Now

$g_w(0) = 0$ when each of the fuzzy value for the set A and B are equal.

and

$g_w(1) = 0$ when each of the fuzzy value for the set A and B are equal.

Also

$$g'_w(0) = \sum_{i=1}^{n} w_i\left[\mu_A(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)}+(1-\mu_A(x_i))\log\frac{1-\mu_A(x_i)}{1-\mu_B(x_i)}\right]\geq 0$$

and

$$g'_w(1)=\sum_{i=1}^{n} w_i \left[\mu_B(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)} + (1-\mu_B(x_i))\log\frac{1-\mu_A(x_i)}{1-\mu_B(x_i)}\right] \le 0$$

Next, we present the preceding mentioned information in the following Table-2.1 for building the meticulous judgment.

Table-2.1

λ	w_i	$\mu_A(x_i)$	$\mu_B(x_i)$	$g_w(\lambda)$
	2	0.2	0.1	
	3	0.3	0.3	
	4	0.4	0.5	
0.1	5	0.5	0.3	0.14
	6	0.4	0.1	
	7	0.3	0.2	
	2	0.2	0.1	
	3	0.3	0.3	
	4	0.4	0.5	
0.2	5	0.5	0.3	1.25
	6	0.4	0.1	
	7	0.3	0.2	
	2	0.2	0.1	
	3	0.3	0.3	
	4	0.4	0.5	
0.3	5	0.5	0.3	2.15
	6	0.4	0.1	
	7	0.3	0.2	
	2	0.2	0.1	
	3	0.3	0.3	
	4	0.4	0.5	
0.4	5	0.5	0.3	2.81
	6	0.4	0.1	
	7	0.3	0.2	
	2	0.2	0.1	
	3	0.3	0.3	
	4	0.4	0.5	

0.5	5	0.5	0.3	2.7998
	6	0.4	0.1	
	7	0.3	0.2	
0.6	2	0.2	0.1	1.52
	3	0.3	0.3	
	4	0.4	0.5	
	5	0.5	0.3	
	6	0.4	0.1	
	7	0.3	0.2	
0.7	2	0.2	0.1	1.27
	3	0.3	0.3	
	4	0.4	0.5	
	5	0.5	0.3	
	6	0.4	0.1	
	7	0.3	0.2	
0.8	2	0.2	0.1	0.12
	3	0.3	0.3	
	4	0.4	0.5	
	5	0.5	0.3	
	6	0.4	0.1	
	7	0.3	0.2	

Graphically $g_w(\lambda)$

provides the following figure:

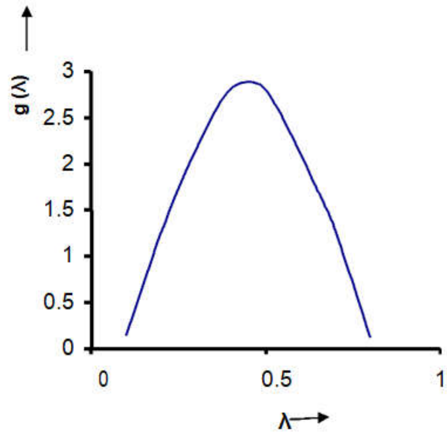

Fig-2.1

Next, we scrutinize that $g_w(\lambda)$ gives maximum at λ^* when

$$\sum_{i=1}^{n} w_i\left[\mu_A(x_i)\log\mu_A(x_i)+(1-\mu_A(x_i))\log(1-\mu_A(x_i))\right]$$

$$-\sum_{i=1}^{n} w_i\left[\mu_B(x_i)\log\mu_B(x_i)+(1-\mu_B(x_i))\log(1-\mu_B(x_i))\right]$$

$$=\sum_{i=1}^{n} w_i\{\mu_A(x_i)-\mu_B(x_i)\}\log\frac{\lambda^*\mu_A(x_i)+(1-\lambda^*)\mu_B(x_i)}{\lambda^*(1-\mu_A(x_i))+(1-\lambda^*)(1-\mu_B(x_i))} \qquad (2.2)$$

Let $\lambda^* A+(1-\lambda^*)B$ be denoted by C. Then the above equation (2.2) proves that

$$D_1(A:C;W) = D_1(B:C;W)$$

This appearance verifies that the generalized model is maximum for that distribution which is in a number of means halfway in sandwiched between A and B .

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