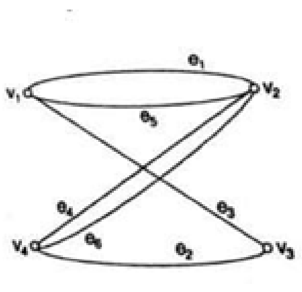
GROWTH IN THE DOMAIN OF MATHEMATICAL STUDY ON THE RELATION BETWEEN COMPLETE GRAPH AND EULER CIRCIUT

[1]Ravi Padma, [2]G.Sampath Kumar, [3]A. Chandra Sekhar
[1,2,3]Assistant Professor, [1,2,3]Department of H & S, Brilliant Institute of Engineering and Technology, Hyderabad, India

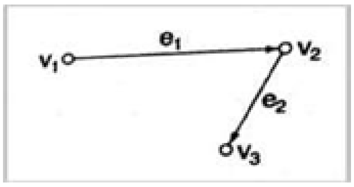
ABSTRACT
Mathematicians who specialize in graph theory study how networks and graphs are built. Since the concepts are entirely broad, graphs can be applied to a wide range of application domains, provided that a suitable representation can be determined. Nonetheless, it has developed into an important area of mathematical study with applications in a number of disciplines, including chemistry, biology, sociology, computer science, etc....
Keywords: Graph, Connected graph, Euler circuit, complete graph.

INTRODUCTION
The term "Eulerian" relates to Leonhard Euler, a Swiss mathematician who developed graph theory in the 18th century. He gained notoriety for resolving the issues with the Konigsberg Bridge. He founded the field of graph theory. We analyze the relationship between complete graphs and Euler graphs in this essay.

DEFINITIONS
Definition 1
A Graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges denoted by V(G) and E(G) respectively.
If e={u,v} is an edge, we write e=uv; we say that e joins the vertices u and v; u and v are adjacentvertices;u and v are incident with e.

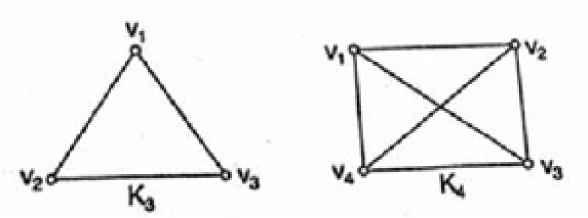


Definition 2
A graph G is said to be connected if any two distinct vertices of G are joined by a path.

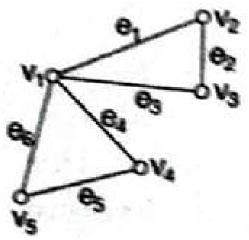


Definition 3

A simple graph with n vertices in which there is an edge between every pair of distinct vertices is called a complete graph on n vertices. This graph is denoted by K_n.



Definition 4



A circuit in a graph that includes all the edges of the graph is called an Euler circuit.

RESULTS

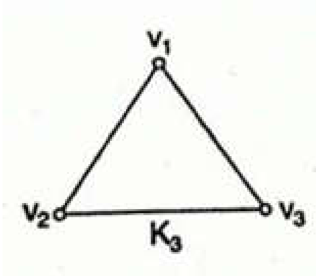
If G is a connected graph and every vertex has even degree,then G has an Euler circuit.
A Graph G be a complete graph with n vertices has n-1 degrees.

THEOREM

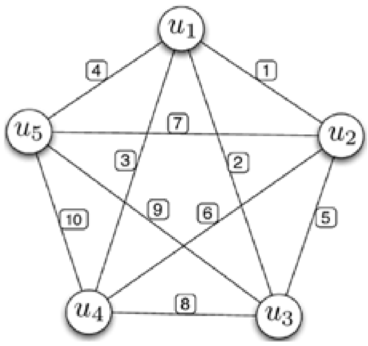
Let G be a connected graph with n vertices and G is also a complete graph the K_n is an Euler circuit if n is odd.
Proof
Let G=(V,E) be a connected graph with n vertices.
This theorem is proved by induction method
Basic Step:
If n=1
The graph G has only vertex with degree 0.
Since the graph G connected graph.
By the definition of Euler circuit, G has a Euler circuit.
Inductive hypothesis:
If n=k-1 is even.
Let Complete graph K_{n-1} is an Euler circuit
Inductive step:
To prove: K is odd
Let G=(V,E) be a connected and complete graph with n vertices.
Then each vertex in a complete graph has n-1 degrees which is even.
This implies k is odd
We know that if G is a connected graph and every vertex has even degree the G has an Euler circuit.
ThereforeK_n has an Euler circuit.
Ilustration 1
If n is Odd:
If n=3

Let K_3 ,V = $\{v_1,v_2,v_3\}$ and E = $\{e_1,e_2,e_3\}$ be a complete graph with 3 vertices.

$d(v_1) = 2$; $d(v_2) = 2$; $d(v_3) = 2$ which is even.

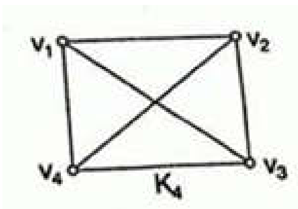


In this graph v_1,e_1,v_2, e_2,v_3,e_3,v_1 is an Euler circuit. Since K_3 has all vertices and all edges exactly ones,
Hence K_3 has an Euler Circuit.
If n=5
Let K_5 ,V = $\{v_1,v_2,v_3, v_4,v_5\}$ and E = $\{e_1,e_2,e_3, e_4,e_5\}$ be a complete graph with 5 vertices.
$d(v_1) = 4$; $d(v_2) = 4$; $d(v_3) = 4$; $d(v_4) = 4$; $d(v_5) = 4$ which is even.



In this graph v_1,e_1,v_2, $e_8,v_5,e_{10},v_3,e_3,v_4$, e_4,v_5,e_5,v_1,e_6, v_4,e_9,v_2,e_2, v_3,e_7,v_1 is an Euler circuit.
Since K_3 has all edges exactly ones.

Hence K_5 has an Euler Circuit.
Ilustration 2
If n is even
If n=4
Let K_4 ,V = $\{v_1,v_2,v_3,v_4\}$ and E = $\{e_1,e_2,e_3,e_4\}$ be a complete graph with 4 vertices.
$d(v_1) = 3$; $d(v_2) = 3$; $d(v_3) = 3$; $d(v_4) = 3$ which is odd.



In this graph v_1,e_1,v_2, e_2,v_3,e_5,v_1,e_4,v_4, e_6,v_2,e_2,v_3,e_3,v_4 which is not an Euler circuit.
Since K_4, the edge e_2 appears twice in the path.
Which is contradiction to our definition of Euler circuit.

USES OF EULER CIRCUIT IN REAL LIFE:

1. Connecting with friends on social media,where each user is a vertex, and when users connect they create an edge.
2. Usin GPS/Google maps/Yahoo maps, to find a route based on shortest route.
3. Delivering the Mail.
4. The Travelling Salesman

CONCLUSION

In this paper, We try to make an attempt of an elaborate study about the significance of relation between complete graph and euler circuit and its practical uses.

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