DUAL PERSON ZERO SUM GAME PREDICAMENT USING PENTAGONALFUZZY NUMBERS

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ABSTRACT
This study examines a two-player, zero-sum game using a fuzzy payout matrix made up of pentagonal fuzzy integers. The fuzzy game problem is transformed into a crisp problem using a newly presented method, and then it is solved using the standard game problem method. Additionally, a comparison between the suggested strategy and the ranking system is done.
Key words- Pentagonal Fuzzy numbers, Fuzzy payoff matrix, Two person zeroSum game, Ranking

INTRODUCTION

The study of decision-making in circumstances where two or more rational adversaries try to maximize their benefit in competitive scenarios is known as game theory. According to this decision-making theory, each player chooses his course of action after taking into account the options that the opponent player has. This theory's goal is to understand how players choose their specific strategies to maximize their payoffs. John von Neumann, a mathematician, and economist Oskar Morgenstren invented the game theory method. The methodology used by Neumann (1947) [4] is based on the maximization of minimum losses, or the maximization of best out of worst. The fuzzy set theory was initially introduced by Zadeh in 1965 [6]. This concept of fuzzy decision that may be viewed as the intersection of given fuzzy goals and / or fuzzy constraints was illustrated by Bellman and Zadeh (1970) [1]. M. Cevikel
A.C & Ahlatcioglu.M (2010) [3] presented two models for studying two persons zero sum matrix games in which the payoffs and goals are fuzzy and obtained that the fuzzy relation approach and and the max-min solution are equivalent. Li Deng-Feng (2012)
[2] developed an effective method for solving games with payoffs of triangular fuzzy numbers. Selva kumari. K & Lavanya. S at (2015) [5] considered a two zero sum gamewith imprecise (triangular or trapezoidal) fuzzy number using ranking function for an approach to solve the problem.
In the current paper, we have proposed an approach for solving the fuzzy gameproblem by converting to its equivalent crisp form.
The rest of this paper is organized as follows. In section 2, we recall the basic definitions, in section 3, elaborates the mathematical formulation of fuzzy game problem we have proposed the solution for solving fuzzy game problem using pentagonal fuzzy numbers. In section 4, Numerical example is provided to illustrate the efficiency of the proposed method. Section 5 gives the conclusion of this Paper

Preliminaries

Fuzzy Numbers

A fuzzy range may be a generalization of an everyday complex quantity and that doesn't confer with one worth however rather to a connected a group of attainable worth, wherever every attainable worth has its weight between zero and one.
A fuzzy number is a convex normalized fuzzy set on the real line R such thatThere exist at least one

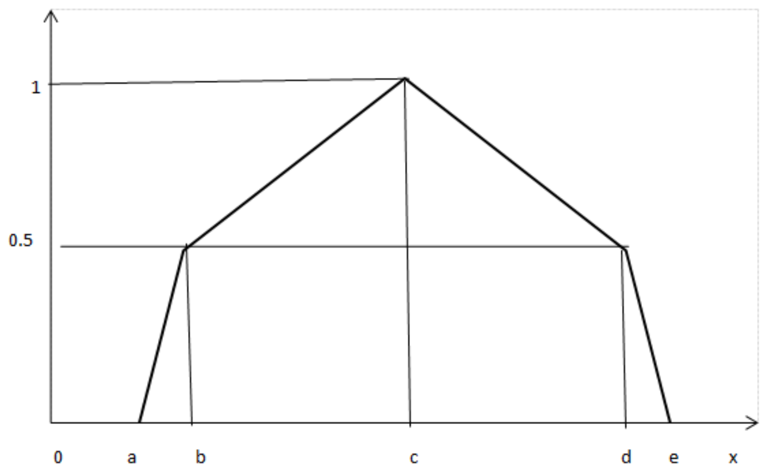
i) x€ X with $\mu_A(X)$ =1
ii) $\mu_A(X)$ is a piecewise continuous.

Pentagonal Fuzzy Number

A fuzzy number $\tilde{A}$ is said to be Pentagonal fuzzy number $\tilde{A}_P$ (a, b, c, d, e) where a, b, c, d, e are real numbers and its membership is given by

$$\mu_{\tilde{A}_p}(x) = \begin{cases} \frac{1}{2}\frac{(x-a)}{(b-a)} & for\ a \le x \le b \\ \frac{1}{2} + \frac{1}{2}\frac{(x-b)}{(c-b)} & for\ b \le x \le c \\ 1 - \frac{1}{2}\frac{(x-c)}{(d-c)} & for\ c \le x \le d \\ \frac{1}{2}\frac{(c-x)}{(e-d)} & for\ d \le x \le e \end{cases}$$

Figure 1. Graphical Representation of Pentagonal Fuzzy Number



RANKING FUNCTION

A ranking function of a fuzzy number R: F(R) →R is the set of all fuzzy numbers defined on the set of Real Numbers, which maps each fuzzy number into a real number. Here F(R) denotes the set of all pentagonal fuzzy numbers defined on R.

Ranking a fuzzy number involves measuring up to two fuzzy numbers, and defuzzification is a technique whereby the fuzzy number is renewed to an approximated crisp number.

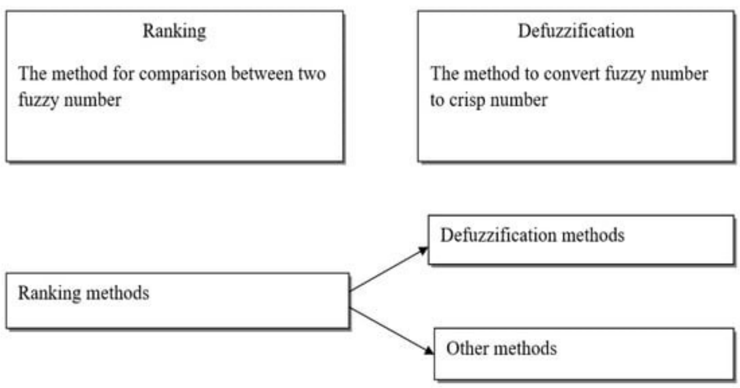


Figure 2. Defuzzification and Ranking concept.

RANKING FORMULA OF PENTAGONAL FUZZY NUMBERS

$$R(A_P)=\frac{2a+3b+2c+3d+2e}{4}$$

Mathematical Formulation of Fuzzy Game problem

A finite two-person zero-sum game which is represented in matrix form is called a matrix game. This is a direct consequence of the fact that two opponents with exactlyopposite interests play a game under a finite number of strategies, independently of his or her opponent's action. Once both players each make an action, their decisionsare disclosed. A payment is made from one player to the other based on the outcome,such that the gain of one player equals the loss of the other, resulting in a net payoff summing to zero.

The Fuzzy Payoff Matrix

"The problem that we are aiming to solve is a two player zero sum fuzzy game in which the entries in the payoff matrix A are pentagonal fuzzy number. The fuzzy payoffmatrix is

$$\tilde{A}=\begin{bmatrix} \tilde{a}_{11}\,\tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{a}_{m1}\tilde{a}_{m2} & \cdots & \tilde{a}_{mn} \end{bmatrix}$$

P layer A has m strategies and Player B has n strategies. If player A chooses the i^{th} strategies and Player B chooses j^{th} strategies the Player A win.

Saddle point

If the max-min value equals the mini-max value then the game is said to a saddle (equilibrium) point and the corresponding strategies are called optimum strategies. The amount of payoff at an equilibrium point is known as the value of the game.

Pure strategy

Pure strategy is a decision-making rule in which one particular course of action is selected. The course of the fuzzy game is determined by the desire of A to maximizehis gain and that of restrict his loss to a minimum.

Solution for fuzzy game problem using statistical tools

Numerical example

Fuzzy Game with Payoffs as Pentagonal Fuzzy Number

Player B

	(2,4,5,7,10)	(2,4,9,9,10)	(1,5,8,9 10)
Player A	[(1,3,4,6,7)	(2,3,6,8,9)	(1,2,4,7,10)]
	(2,4,5,7,8)	(2,3,7,8,8)	(3,5,6,7,10)

The following table gives the crisp values of fuzzy pentagonal numbers

Strategy	**PentagonalNumbers**	$\sum$**X**	$\bar{X}$	$\sum\frac{\lvert X-\bar{X}\rvert}{N}$

$\tilde{a}_{11}$	(2,4,5,7,8)	26	5.2	1.84
$\tilde{a}_{12}$	((2,3,7,8,8)	28	5.6	2.4
$\tilde{a}_{13}$	(3,5,6,7,10)	31	6.2	1.8
$\tilde{a}_{21}$	(1,3,4,6,7)	21	4.2	1.8
$\tilde{a}_{22}$	(2,3,6,8,9)	28	5.6	2.4
$\tilde{a}_{23}$	(1,2,4,7,10)	24	4.8	2.9
$\tilde{a}_{31}$	(2,4,5,7,10)	28	5.6	2.3
$\tilde{a}_{32}$	(2,4,9,9,10)	34	6.8	3
$\tilde{a}_{33}$	(1,5,8,9,10)	33	6.6	2.8

The given fuzzy game is reduced into crisp game problem using statisticaltools (mean) and solved by maximin- minimax criterion as follows:

$$\begin{pmatrix} 5.2 & 5.6 & 6.2 \\ 4.2 & 5.6 & 4.8 \\ 5.6 & 6.8 & 6.6 \end{pmatrix}$$

To find a saddle point

Minimum of 1st row =5.2

Minimum of 2nd row =4.2

Minimum of 3rd row =5.6

Max(min)=5.6

Maximum of 1st column = 5.6

Maximum of 2nd column = 6.8

Maximum of 3rd column = 6.6

Min(max)=5.6

Max(min)=Min(max)=5.6, saddle point exists.

The fuzzy value of the game is $(\mathbf{2}, \mathbf{4}, \mathbf{5}, \mathbf{7}, \mathbf{8})$

The given fuzzy game is reduced into crisp game problem using statistical tools (mean deviation) and solved by maximin- minimax criterion as follows:

$$\begin{pmatrix} 1.84 & 2.4 & 1.8 \\ 1.8 & 2.4 & 2.9 \\ 2.3 & 3 & 2.8 \end{pmatrix}$$

To find a saddle point

Minimum of 1st row =1.8

Minimum of 2nd row =1.8

Minimum of 3rd row =2.3

Max(min)=2.3

Maximum of 1st column=2.3

Maximum of 2nd column=3

Maximum of 3rd column=2.9

Min(max)=2.3

max(min)=Min(max)=2.3, saddle point exists.

The fuzzy value of the game is $(2, 4, 5, 7, 8)$

Solution for fuzzy game problem using Ranking Method

By applying the ranking formula as in definition 2.4 the fuzzy pay off values areconverted into crisp values as shown in the table below:

Ranking of the Pentagonal fuzzy numbers in the fuzzy game problem

Strategy	Pentagonal Numbers	Ranking Function $R(\tilde{A}) = \frac{1}{4}(2a+3b+2c+3d+2e)$
$\tilde{a}_{11}$	(2,4,5,7,8)	$R(\tilde{a}_{11})$ = 15.75
$\tilde{a}_{12}$	((2,3,7,8,8)	$R(\tilde{a}_{12})$ = 16.75
$\tilde{a}_{13}$	(3,5,6,7,10)	$R(\tilde{a}_{13})$ = 18.5
$\tilde{a}_{21}$	(1,3,4,6,7)	$R(\tilde{a}_{21})$ = 12.75
$\tilde{a}_{22}$	(2,3,6,8,9)	$R(\tilde{a}_{22})$ = 16.75
$\tilde{a}_{23}$	(1,2,4,7,10)	$R(\tilde{a}_{23})$ = 14.25
$\tilde{a}_{31}$	(2,4,5,7,10)	$R(\tilde{a}_{31})$ = 16.75
$\tilde{a}_{32}$	(2,4,9,9,10)	$R(\tilde{a}_{32})$ = 20.25
$\tilde{a}_{33}$	(1,5,8,9,10)	$R(\tilde{a}_{33})$ = 20

The given fuzzy game is reduced into crisp game problem and then solved bymaximin-minimax criterion as follows:

$$\begin{pmatrix} 15.75 & 16.75 & 18.5 \\ 12.75 & 16.75 & 14.25 \end{pmatrix}$$

$$16.75 \quad 20.25 \quad 20$$

To find a saddle point
Minimum of 1st row =15.75
Minimum of 2nd row =12.75
Minimum of 3rd row =16.75

Max(min)=16.75
Maximum of 1st column = 16.75
Maximum of 2nd column = 20.25
Maximum of 3rd column = 20.00

Min(max)=16.75
Max(min)=Min(max)=16.75, saddle point exists.

The fuzzy value of the game is $(\mathbf{2}, \mathbf{4}, \mathbf{5}, \mathbf{7}, \mathbf{8})$

Conclusion

In this study, we found the best solution to a fuzzy game utilizing pentagonal fuzzy integers as rewards. By using the Ranking method and statistical techniques, the fuzzy payoffs are transformed into crisp numbers. The aforementioned illustration clearly demonstrates that the suggested approach may provide a fuzzy solution for the fuzzy matrix game.

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