A BRIEF COMMENT ON OPERATIONS ON CLOSED INTERVALS BASED ON FUZZY ARITHMETIC BASED OPERATIONS

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Abstract
The topics covered in this paper include fuzzy sets, fuzzy set support, fuzzy set intersection, and fuzzy arithmetic-based operations on closed intervals. The Study also explains how these Operations are helpful in Graduation Issues and Fuzzy Quadratic Programming Challenges. The study will be helpful for research purposes as well as for teachers and students in mathematics classrooms.
Key Words : Fuzzy Set, Convex Fuzzy Set, Crisp ,Fuzzy Arithmetic.

Introduction
One of the primary and most significant problems that contemporary management systems must face is uncertainty. The principal topics of contemporary analysis share a number of common characteristics that make them particularly challenging for current methodologies. Complexity, dynamics, and unpredictability are these characteristics. The usual procedures may not be sufficient in some circumstances because of the uncertainty [1]. It is crucial for decision-makers to take into account and assess the uncertainty associated with the problem in question as well as its surrounds when it comes time to make a wise choice about an unclear situation. Several factors can cause uncertainty, including incomplete or hazy understanding of future circumstances, erroneous data, incorrect forecasting, personal preferences, and the presence of uncontrollable outside disruptions. To make decisions in unclear circumstances, one needs often establish active approach rather than ignore it.
Classical set theory based on two-valued logic defines a set as a collection of objects with well-defined 'crisp' boundaries. An element either belongs to the set or does not belong to the set, that is, its membership is either 1 or 0. To deal with the sets with imprecise boundaries, Lotfi A. Zadeh [2] in 1965 introduced fuzzy set theory. The membership function in a fuzzy set, unlike that of a 'crisp' set is a not matter of being either true or false, but a matter of degree of truth/belief. In general, degrees of membership in fuzzy sets are expressed by values in [0, 1]. The extreme values 0 and 1 in the interval [0, 1] represent total non-belongingness and total belongingness respectively.
This makes, crisp sets, a special case of fuzzy sets, for which only two grades of memberships are allowed. Thus, we can say 'crisp is fuzzy', with a membership of either 1 or 0.
Probability theory is the traditional theory describing and measuring the phenomenon of uncertainty. It is assumed that the probability theory can be used in every situation of uncertainty [3]. Since both fuzzy set theory and probability theory deal with uncertainty, most of the time former is confused with later. But fuzziness is only one aspect of uncertainty. It is the vagueness or ambiguity found in the definition of a concept or meaning of terms. The probability generally relates to randomly occurring events that are clearly defined and may contain the uncertainty of randomness.
Fuzzy logic is basically a multi valued logic that allows intermediate values to be defined between conventional evaluations like yes/no, true/false, tall/very tall, etc. Fuzzy reasoning and logic have the ability to express the amount of ambiguity in human thinking and subjectivity in a comparatively undistorted manner.

Hence, fuzzy logic techniques find their major applications in areas such as control, pattern recognition, quantitative analysis, inference, and in information retrieval.

Fuzzy systems are being used in various consumer products e.g. washing machines, air conditioners, camcorders, auto-focus cameras, system of traffic light controlling, and subways trains[4].

Fuzzy Set Theory

In this section we introduce some of the basic concepts and terminology of fuzzy set theory.

Theory of fuzzy sets is basically a theory of graded concepts [5].

Fuzzy Set Let X be a classical set of objects, called the universe, whose generic elements are denoted by x. The membership in a crisp subset of X is viewed as characteristic function μA from X to {0, 1} such that:

$$\mu_A(x) = \{ \begin{matrix} 0 & if & x & \notin & A \\ 1 & if & x & \in & A \end{matrix}$$

where {0, 1} is called a valuation set [6].

If the valuation set is allowed to be the real interval [0, 1], A is called a fuzzy set proposed by Zadeh [7]. μ_A (x) is the degree of membership of x in A. The closer the value of μ_A (x) is to l, the more x belongs to A. Therefore, A is completely characterized by the set of ordered pairs:

$$A=\{(x, \mu_A(x)) \mid x \in X\}$$

Where μ_A (x) maps X to the membership space [0, 1]. Elements with zero degree of membership are usually not listed. If Sup $\mu_A(x) = 1, \forall x \in R$, then the fuzzy set A is called a normal fuzzy set in R. A fuzzy set that is not normal is called subnormal fuzzy set.

Support of a Fuzzy Set

The support of a fuzzy set A is a set S(A) such that $x \in S(A) \Leftrightarrow \mu_A(x) > 0$. If $\mu_A(x)$ is constant over S(A), then A is non-fuzzy.

Intersection of Fuzzy Sets

Intersection of two fuzzy sets A and B is a fuzzy set C denoted by C = A B, whose membership function is related to those of A and B by

$$\mu_C(x) = \min[\mu_A(x), \mu_B(x)], \forall x \in X$$

Algebraic Operations on Fuzzy Sets

In addition to the set theoretic operations, we can also define a number of other ways of forming combinations of fuzzy sets and relating them to one another. Here we present some more important operations among those:

1. Algebraic product of two fuzzy sets A and B, is A(.)B, whose membership function is

$$\mu_{A(.)B}(x) = \mu_A(x)(.)\,\mu_B(x), \forall x \in X$$

2. The algebraic sum of A and B is A+B whose membership function is defined as

$$\mu_{(A+B)}(x) = \mu_A(x)(+)\,\mu_B(x), \forall x \in X$$

$$\textit{Provided} \quad \mu_A(x)(+)\,\mu_B(x) \le 1, \forall x \in X$$

Fuzzy Arithmetic

The first definition of a fuzzy set allows us to extend various properties of crisp sets and operations on crisp sets to their fuzzy counterparts.

An ordinary number ‘a’ can be characterized by using the notation of membership function as,

$$\mu_A(x) = \begin{cases} 1 & if \quad x = a \\ 0 & if \quad x \neq a \end{cases}$$

A fuzzy number A is a fuzzy set on the real line R that possesses the following properties:

A is a normal, convex fuzzy set on R,

The a-level set A $_a$ must be a closed interval for every α□ [0, 1],

The support of A, S(A) = {x|□□$_A$ (x)>0}, must be bounded.

Fuzzy arithmetic is based on two properties of fuzzy numbers:

□ Each fuzzy set and thus, each fuzzy number can be fully and uniquely represented by its α-level sets.

α-level sets of each fuzzy numbers are closed intervals of real numbers for all α□□
[0, 1].

These properties enable use to define an arithmetic operation on fuzzy numbers in terms of arithmetic operations on their a-level sets (i.e. arithmetic operations on closed intervals).

Fuzzy Arithmetic Based Operations on Closed Intervals

A fuzzy number can be characterized by an interval of confidence at level α,

$$A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}]$$

which has the property

$$\alpha \le \alpha' \Rightarrow A_{\alpha'} \subset A_\alpha$$

Let A = [a, b]□□ *R* and B = [c, d]□□ *R* be two fuzzy numbers then we define the arithmetic operations on them as

Addition	A+B = [a + c, b + d]
Subtraction	A-B = [a – d, b – c]
Multiplication	AB = [min (ac, ad, bc, bd), max (ac, ad, bc, bd)]
Inverse of A	A^{-1} = [min (1/a, 1/b), max (1/a, 1/b)]
Division	A/B = [min (a/c, a/d, b/d), max (a/c, a/d, b/c, b/d)]

Minimum ($\wedge$)	$A \wedge B = [a \wedge c, b \wedge d]$
Maximum ($\vee$)	$A \vee B = [a \vee c, b \vee d]$

Let A and B he two fuzzy numbers, $A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}]$ be the α-level set of A, and $B_\alpha = [b_1^{(\alpha)}, b_2^{(\alpha)}]$ be the α-level set of B.

Let * denote any of the arithmetic operations +, -, ., , /, $\wedge$ *and* $\vee$ on fuzzy numbers.

Then, we define a fuzzy set A* B in R, by defining its α-level sets $(A * B)_\alpha$ as

$$(A * B)_\alpha = A_\alpha \text{ for any } \alpha \in [0,1]$$

Since $(A * B)_\alpha$ is a closed interval for each $\alpha \in [0,1]$ and A and B are fuzzy numbers, A * B is also a fuzzy number.

The multiplication of fuzzy number $A \subset R$ by an ordinary number $k \in R^+$ can also be defined as

$(k * A_\alpha) = k\,(.)\,A_\alpha = [ka_1^{(\alpha)}, ka_2^{(\alpha)}]$ or equivalently, $\mu_{k.A}(x) = \mu_A(x/k)\ \forall x \in R$

CONCLUSION

This study concentrated on Operations on Closed Intervals based on Fuzzy Arithmetic. The finding is significant because these operations might be used in future studies on graduation problems and fuzzy quadratic programming problems.

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