ENTROPIC MODEL GENERATION FOR FUZZY DISTRIBUTIONS USING DISTANCE MEASURES

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ABSTRACT

The two fundamental ideas of measures of entropy and directed divergence, which are a subset of applied mathematics, are extremely important and play a big part in the research on application domains, in information theory. By utilizing the idea of divergence models, we have created various new entropic models for fuzzy distributions and fashioned the relationships between the two in the current communication.

Keywords: Uncertainty, Fuzzy distribution, Fuzzy set, Fuzzy entropy, Crisp set, Concavity, Fuzzy divergence.

INTRODUCTION

In situations when precise research is either difficult or impractical, it is a well-known observable truth that fuzzy set theory is a powerful tool for modeling undefined and indistinguishable circumstances. This notion was given by Zadeh [20] to help with the set's fuzziness. The text of information measures contains many quantitative fuzzy entropic models. According on Shannon's [19] entropy, De Luca and Termini [3] suggested the fuzzy entropic model described below:

$$H(A) = -\sum_{i=1}^{n} [\mu_A(x_i)\log \mu_A(x_i) + (1-\mu_A(x_i))\log(1-\mu_A(x_i))] \qquad (1.1)$$

Bhandari and Pal [2] recommended the following fuzzy entropic model:

$$H_\alpha(A) = \frac{1}{1-\alpha}\sum_{i=1}^{n} \log\left\{\mu_A^\alpha(x_i) + (1-\mu_A(x_i))^\alpha\right\}; \alpha \neq 1,\ \alpha > 0 \qquad (1.2)$$

Kapur [8] outlined the following fuzzy entropic model:

$$H^\alpha(A) = (1-\alpha)^{-1}\sum_{i=1}^{n}\left[\left\{\mu_A^\alpha(x_i) + (1-\mu_A(x_i))^\alpha\right\} - 1\right]; \alpha \neq 1, \alpha > 0 \qquad (1.3)$$

The model (1.3) corresponds to Havrada and Charvat's [4] entropic model.

Parkash [14] outlined the following new generalized model involving two real parameters and studied its many interesting properties.

$$H_\alpha^\beta(A) = [(1-\alpha)\beta]^{-1} \sum_{i=1}^{n} \left[\left\{ \mu_A^\alpha(x_i) + (1-\mu_A(x_i))^\alpha \right\}^\beta - 1 \right], \alpha>0, \quad \alpha \neq 1, \beta > 0 \qquad (1.4)$$

Parkash and Sharma [15] shaped several measures of fuzzy entropy, obtained some interaction among these measures and applied them to an assortment of disciplines.

On the other hand, Bhandari and Pal [2] profiled the following distance model between two fuzzy sets and it corresponds to Kullback and Liebler's [10] divergence model:

$$D(A:B) = \sum_{i=1}^{n} \left[\mu_A(x_i) \log \frac{\mu_A(x_i)}{\mu_B(x_i)} + (1-\mu_A(x_i)) \log \frac{1-\mu_A(x_i)}{1-\mu_B(x_i)} \right] \qquad (1.5)$$

Kapur [8] has outlined countless fascinating mathematical expressions of fuzzy entropic models corresponding to Kapur's [7] probabilistic models. Countless other developments regarding the revision of divergence models for fuzzy distributions has been made by Kapur [8], Parkash [14], Joshi and Kumar [5, 6], Markechová, Mosapour and Ebrahimzadeh [12], Kobza [9] etc.

To generate new entropic models, we reflect on the following function:

(1.6)

The result (1.6) can be second-handed to shape a fuzzy entropic model on behalf of each divergence model. Some other possibilities can be taken as follows:

$$H_2(A) = D^a(C:F) - D^a(A:F) \qquad (1.7)$$

$$H_3(A) = e^{D(C:F)} - e^{D(A:F)} \qquad (1.8)$$

$$H_4(A) = e^{-D(A:F)} - e^{-D(C:F)} \qquad (1.9)$$

$$H_5(A) = \log \frac{D(C:F)}{D(A:F)} \qquad (1.10)$$

$$H_6(A) = \sum_{i=1}^{n} \mu_A(x_i) D(A:C) \qquad (1.11)$$

A quantity of efforts related with the continuity of fuzzy entropic models has been made by Bassanezi and Roman-Flores [1] while lots of other fuzzy models have been originated by De Luca-Termini [3], Kapur [8], Parkash [14], Bhandari and Pal [2], Parkash, Sharma and Mahajan [16, 17], Li and Liu [11], Osman, Abdel-Fadel, El-Sersy and Ibraheem [13] etc.

2. DEVELOPMENT OF MEASURES OF FUZZY ENTROPY

Taking into consideration the functions proposed above, we derive the following fuzzy entropic model:

(a) Measures of fuzzy entropy based on Bhandari and Pal's [2] model:

The divergence model analogous to Bhandari Kullback-Leibler's [9] model was provided by and Pal's [2] and is given by

$$D(A:B) = \sum_{i=1}^{n}\left[\mu_A(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)} \pm (1-\mu_A(x_i))\log\frac{1-\mu_A(x_i)}{1-\mu_B(x_i)}\right] \quad (2.1)$$

(i) Using (2.1), the relation (1.6) gives

$$H_1(A) = D(C:F) - D(\mu_A(x_i):F)$$

$$= n\log 2 - \sum_{i=1}^{n}\left[\mu_A(x_i)\log\mu_A(x_i) + (1-\mu_A(x_i))\log(1-\mu_A(x_i))\right]$$

$$- \sum_{i=1}^{n}\left[\mu_A(x_i)\log 2 + (1-\mu_A(x_i))\log 2\right]$$

$= H(A)$ which is fuzzy entropic model outlined by De Luca and Termini [3].

(ii) Using (2.1), the relation (1.7) gives

$$H_2(A) = (n\log 2)^a - \left[\sum_{i=1}^{n}\left[\mu_A(x_i)\log(2\mu_A(x_i)) + (1-\mu_A(x_i))\log(2(1-\mu_A(x_i)))\right]\right]^a; \ a>1$$

$$= (n\log 2)^a - \left[-H(A) + n\log 2\right]^a$$

It can be proved that the expression obtained above verifies each and every one property and hence can be taken a new fuzzy entropic model.

(iii) Using (2.1), the relation (1.8) gives

$$H_3(A) = e^{n\log 2} - e^{(-H(A)+n\log 2)} = 2^n\left[1-e^{-H(A)}\right]$$

which can be taken as a new fuzzy entropic model.

(iv) Using (2.1), the relation (1.9) gives

$$H_4(A) = e^{-(-H(A)+ \dots)} = 1\left[e^{H(A)}\right.$$

$\overline{2^{n}}$

$n \log 2)$ -

$e^{-n \log 2}$

$-1]$ which can be taken as a new fuzzy entropic model.

(v) Using (2.1), the relation (1.10) gives

$$H_5(A) = \log \frac{n \log 2}{H(A) + n \log 2}$$

$$= -\log\left[1 - \frac{H(A)}{n \log 2}\right]$$ which is again a new fuzzy entropic model.

(vi) Using (2.1), the relation (1.11) gives

$$H_6(A) = -\sum_{i=1}^{n} \mu_A(x_i) \log \mu_A(x_i) - \sum_{i=1}^{n} \mu_A(x_i) \log(1 - \mu_A(x_i))$$

which can be taken as a new fuzzy entropic model.

(b) Measures of fuzzy entropy based on Kapur's [8] measure:

The fuzzy divergence equivalent to Havrada-Charvat's [4] model was provided by Kapur's [8] and is given by

$$D^{\alpha}(A:B) = \frac{1}{\alpha - 1}\sum_{i=1}^{n}\left[\mu_A^{\alpha}(x_i)\mu_B^{1-\alpha}(x_i) + (1 - \mu_A(x_i))^{\alpha}(1 - \mu_B(x_i))^{1-\alpha} - 1\right] \quad (2.2)$$

(i) Using (2.2), the relation (1.6) gives

$$H_1^{\alpha}(A) = D^{\alpha}(C:F) - D^{\alpha}(A:F) = \frac{n}{\alpha - 1}2^{\alpha-1} + 2^{\alpha-1} H^{\alpha}(A)$$

where $H^{\alpha}(A) = \sum_{i=1}^{n}\left[\mu_A^{\alpha}(x_i) + (1 - \mu_A(x_i))^{\alpha} - 1\right]$ is a fuzzy entropic model and it

Thus

$$H_1^{\alpha}(A)=2^{\alpha-1}\left[H^{\alpha}(A)+\frac{n}{\alpha-1}\right]$$ can be taken as a new fuzzy entropic model.

(ii) Using (2.2), the relation (1.7) gives

$$H_2^{\alpha}(A)=\frac{n}{(\alpha-1)^{a}}\left(2^{\alpha}-1\right)^{a}-\frac{1}{(\alpha-1)^{a}}2^{(\alpha-1)a}\left[\sum_{i=1}^{n}\left\{\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}-2^{1-\alpha}\right\}\right]^{a}$$

$$=\frac{n}{(\alpha-1)^{a}}\left(2^{\alpha}-1\right)^{a}-\frac{2^{(\alpha-1)a}}{(\alpha-1)^{a}}\left[(1-\alpha)H^{\alpha}(A)+n\left(1-2^{1-\alpha}\right)\right]^{a}$$

which can be taken as new fuzzy entropic model.

(iii) Using (2.2), the relation (1.8) gives

$$H_3^{\alpha}(A)=\exp.\left\{\frac{n}{(\alpha-1)}\left(2^{\alpha}-1\right)\right\}-\exp.\left[2^{\alpha-1}\left\{-H^{\alpha}(A)+\frac{n}{(\alpha-1)}-\frac{n2^{1-\alpha}}{(\alpha-1)}\right\}\right]$$

which is a new fuzzy entropic model.

(iv) Using (2.2), the relation (1.9) gives

$$H_4^{\alpha}(A)=\exp.\left[2^{\alpha-1}\left\{H^{\alpha}(A)-\frac{n}{(\alpha-1)}+\frac{n2^{1-\alpha}}{(\alpha-1)}\right\}\right]-\exp.\left\{\frac{n}{(\alpha-1)}\left(1-2^{\alpha}\right)\right\}$$

which can be taken as new fuzzy entropic model.

(v) Using (2.2), the relation (1.10) gives

$$H_5^{\alpha}(A)=\log.\left\{\frac{n\left(2^{\alpha}-1\right)}{2^{\alpha-1}\left\{(1-\alpha)H^{\alpha}(A)+n\left(1-2^{1-\alpha}\right)\right\}}\right\}$$

which is again a new fuzzy entropic model.

(vi) Using (2.2), the relation (1.11) gives

$$H_6^{\alpha}(A)=\frac{1}{\alpha-1}\sum_{i=1}^{n}\mu_A(x_i)\left[\mu_A^{1-\alpha}(x_i)+(1-\mu_A(x_i))^{1-\alpha}-1\right]$$

which can be taken as new model of fuzzy entropy.

(c) Measures of fuzzy entropy based on Kapur's [8] directed divergence:

The fuzzy divergence corresponding to Renyi's [18] divergence was provided by Kapur [8] and is given by

$$D_\alpha(A:B) = \frac{1}{\alpha-1}\sum_{i=1}^{n}\log\left[\mu_A^\alpha(x_i)\,\mu_B^{1-\alpha}(x_i)+(1-\mu_A(x_i))^\alpha(1-\mu_B(x_i))^{1-\alpha}\right] \qquad (2.3)$$

(i) Using (2.3), the relation (1.6) gives

$$_1H_\alpha(A) = \frac{\alpha}{\alpha-1}n\log 2 - \frac{1}{\alpha-1}\sum_{i=1}^{n}\log\left(2^{\alpha-1}\left[\mu_A^\alpha(x_i)+(1-\mu_A(x_i))^\alpha\right]\right)$$

$$= \frac{\alpha\, n\log 2 - n\log 2}{\alpha-1} + H_\alpha(A) = \frac{n\log 2}{\alpha-1} + H_\alpha(A)$$

where $H_\alpha(A)$ is fuzzy entropic model matching to Renyi's [18] model. Thus can be $_1H_\alpha(A)$ taken as a new model of fuzzy entropy.

(ii) Using (2.3), the relation (1.7) gives

$$_2H_\alpha(A) = \left(\frac{\alpha}{\alpha-1}n\log 2\right)^a - \left[n\log 2 - H_\alpha(A)\right]^a$$

$$= (n\log 2)^a\left[\left(\frac{\alpha}{\alpha-1}\right)^a - \left(1-\frac{H_\alpha(A)}{n\log 2}\right)^a\right]$$

Thus $_2H_\alpha(A)$ is a new model of fuzzy entropy.

(iii) Using (2.3), the relation (1.8) gives

$$_3H_\alpha(A) = e^{\frac{\alpha}{\alpha-1}\log 2^n} - e^{n\log 2 - H_\alpha(A)}$$

$$= 2^{\frac{n\alpha}{\alpha-1}} - e^{\log 2^n}e^{-H_\alpha(A)} = 2^{\frac{n\alpha}{\alpha-1}} - 2^n e^{-H_\alpha(A)}$$

which is again a new model of fuzzy entropy.

(iv) Using (2.3), the relation (1.9) gives

$_4H_\alpha(A) = 2^{-n}e^{H_\alpha(A)} - 2^{-\frac{n\alpha}{\alpha-1}}$ which can be taken as new model of fuzzy entropy.

(v) Using (2.3), the relation (1.10) gives

$$_5H_\alpha(A) = \log\left(\frac{\alpha}{\alpha-1}n\log 2\right) - \log\left(n\log 2 - H_\alpha(A)\right)$$

$$= \log\left(\frac{\alpha}{(\alpha-1)\left\{1-\frac{H_\alpha(A)}{n\log 2}\right\}}\right)$$ which is again a new fuzzy entropic model.

(vi) Using (2.3), the relation (1.11) gives

$${}_6H_\alpha(A) = \frac{1}{\alpha-1}\sum_{i=1}^{n}\mu_A(x_i)\left[\log\left\{\mu_A^{1-\alpha}(x_i)+\left(1-\mu_A(x_i)\right)^{1-\alpha}\right\}\right]$$ which can be taken as new fuzzy entropic model.

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