

METHOD OF APPROACHING MULTI OBJECTIVE LINEAR PROGRAMMING PROBLEM WITH BASIC CONCEPT OF OPTIMIZATION TECHNIQUE

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Abstract

In this paper we present some approaches of methods in multi-objective linear programming in different field of Integer programming the basis concept of these available exact methods.

Keyword: Multi-Objective Integer Linear Programming, Optimization Techniques.

Introduction

Involve more than one objective function that are to be minimized or maximized. Answer is set of solutions that define the best trade-off between competing objectives.

PRELIMINARIES, AND PROBLEM FORMULATION

In this section, we extend and introduce some necessary notation and concept related to MOILPs to facilitate presentation and discussion of other sections. Let c^1 and c^2 be n -vectors. A be an $m \times n$ matrix, and b be an m -vector, a MOILP can be started as follows:

$$\max^* z^1(x), z^2(x) \dots z^n(x), \\ x \in K$$

Where: $\{x \in \mathbb{R}^n : Ax \leq b\}$ represent the feasible set in the decision space, and $z_1(x) := c^1 x$ and $z_2(x) := c^2 x$ are two linear objective functions. Note that $\mathbb{R}^n := \{s \in \mathbb{R}^n : s \geq 0\}$. The image Y of X under vector-valued function $z = (z_1, z_2)$ represent the feasible set in the objective / criterion space, i.e., $Y := z(X) := \{y \in \mathbb{R}^2 : y = z(x) \text{ for some } x \in X\}$. It is assumed that X is bounded, and all coefficients / parameters are integer, i.e., $A \in \mathbb{Z}^{m \times n}$, $b \in \mathbb{Z}^m$. $c^i \in \mathbb{Z}^n$ for $i = 1, 2, \dots, n$.

DOMINANCE:

In the single-objective optimization problem, the superiority of a solution over other solutions is easily determined by comparing their objective function values.

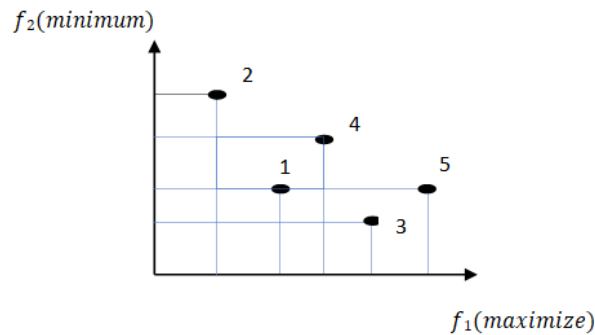
In multi-objective optimization problem, the goodness of a solution is determined by the dominance.

DEFINITION OF DOMINANCE:

Dominance Test: x_1 dominates x_2 , if Solution x_1 is no worse than x_2 in all Objectives. Solution x_1 is strictly better than x_2 in at least one objective.

x_1 dominates $x_2 \leftrightarrow x_2$ is dominated by x_1 .

Example of Dominance Test:



- 1 Vs 2: 1 dominates 2
- 1 Vs 5: 5 dominates 1
- 1 Vs 4: Neither solution dominates.

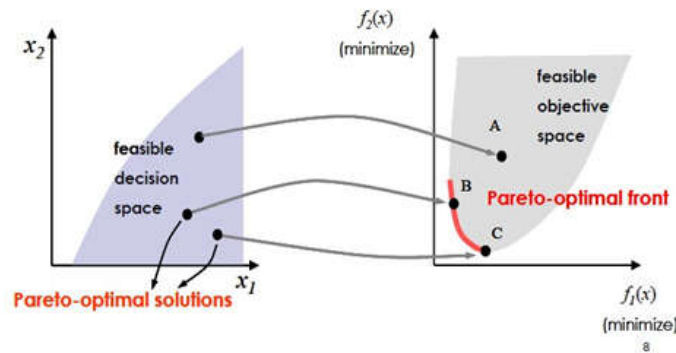
PARETO OPTIMAL SOLUTION:

Non-dominated solution set: Given a set of solution, the non-dominated solution set is a set of all the solution that are not dominated by any member of the solution set.

The non-dominated set of the entire feasible decision space is called the Pareto Optimal set.

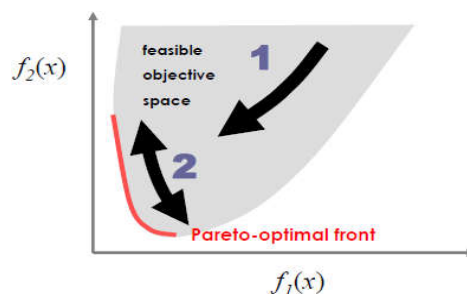
The boundary defined by the set of all point mapped from the Pareto optimal set is called the Pareto optimal front.

GRAPHICAL DEPICTION OF PARETO OPTIMAL SOLUTION:



GOALS IN MULTI-OBJECTIVE OPTIMIZATION:

- Find set of solution as close as possible to Pareto optimal front
- To find a set of solution as diverse as possible



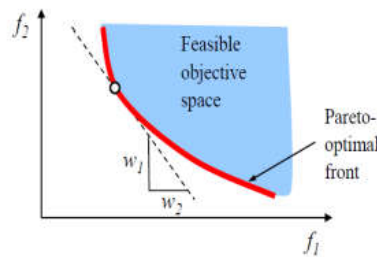
WEIGHTED SUM METHOD

Scalarize a set of objectives into a single objective by adding each objective pre-multiplied by a user-supplied weight.

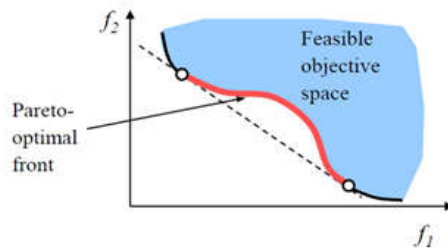
$$\begin{aligned} \text{Minimum } F(x) &= \sum_{m=1}^M w_m f_m(x), \\ \text{Subject to } g_j(x) &\geq 0, \quad j = 1, 2, \dots, j \\ h_k(x) &= 0, \quad k = 1, 2, \dots, k \\ x_i^{(L)} &\leq x_i \leq x_i^{(U)}, \quad i = 1, 2, \dots, n \end{aligned}$$

- ❖ Weight of an objective is chosen in proportion to the relative importance of the objective.
 - ❖ **Advantage:** it is simple
 - ❖ **Disadvantage:** it is difficult to set the weight vectors to obtain a Pareto-optimal solution in a desired region in the objective space.
- It cannot find certain Pareto-optimal solution in the case of a nonconvex objective space.

WEIGHTED SUM METHOD (CONVEX CASE):



WEIGHTED SUM METHOD (CONVEX CASE):



E-CONSTRAINT METHOD

Haimes *et.al.* 1971

It keeps just one of the objectives and restricted the rest of the objective within user-specific values.

Minimum $f_\mu(x)$,

Subject to $f_m(x) \leq s_m$,

$g_j(x) \geq 0$,

$h_k(x) = 0$,

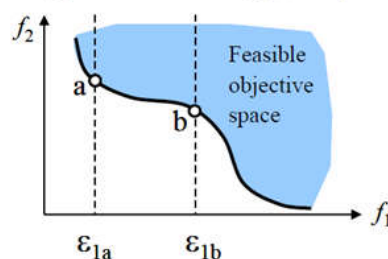
$m = 1, 2, \dots, M$ and $m \neq \mu$

$j = 1, 2, \dots, j$

$k = 1, 2, \dots, k$

Keep f_2 as an objective **Minimize** $f_2(x)$

Treat f_1 as a constraint $f_1(x) \leq \epsilon_1$



Advantage: applicable to either convex or non-convex problems.

Disadvantage: The ϵ vector must be chosen carefully so that it is within the minimum or maximum

values of the individual objective function.

CONCLUSION

In this article paper we conclude that the sort form different methods of multi-objective linear programming in this field of optimization technique.

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